4.Q: Se $f \in L^+$ e $\int f < +\infty$

então

$$\mu(\{x \in X ; f(x) = +\infty\}) = 0$$

RES:

De fato, considere

$$E_n = \{x \in X ; f(x) \geq n\}$$

Note que $n\chi_{E_n} \leq f$

Além disso, note que

$$E_{n+1} = \{x \in X ; f(x) \geq n+1\} \subset \{x \in X ; f(x) \geq n\} = E_n \quad , \forall n$$

e

$$E = \{x \in X ; f(x) = +\infty\} = \bigcap_{n=1}^{\infty} E_n$$

$$\hookrightarrow \int n\chi_{E_n} \leq \int f < \infty$$

$$\Rightarrow n \cdot \mu(E_n) \leq \int f = C < \infty$$

$$\Rightarrow \mu(E_n) \leq \frac{C}{n} \quad , \forall n \in \mathbb{N}$$

Assim $\mu(E) = \mu\left(\bigcap_{n=1}^{\infty} E_n\right) = \lim_{n \to \infty} \mu(E_n) \leq \lim_{n \to \infty} \frac{C}{n} = 0$